Matryoshka

Fast Interactive Verification *through* Strong Higher-Order Automation

Jasmin Christian Blanchette

Education

2000	B.Sc.	Université de Sherbrooke, Canada
2008	M.Sc.	Universitetet i Oslo, Norway
2012	Ph.D.	TU München, Germany

Employment

2000–2008	Software Engineer	Trolltech (Qt), Norway
2008–2014	Research Assistant	TU München, Germany
2015–	Senior Researcher	Inria, France

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Research Area

Proof and counterexample generation in proof assistants

Scientific Impact

10 articles in leading journals (e.g. *J. Autom. Reasoning*, *LMCS*)
26 papers at major conferences (e.g. ICFP, IJCAI, IJCAR, LICS)
1670+ citations on Google Scholar
Best paper award at IJCAR 2016

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PC member of top deduction conferences (CADE, IJCAR, …)

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3 Ph.D. students, 1 postdoc

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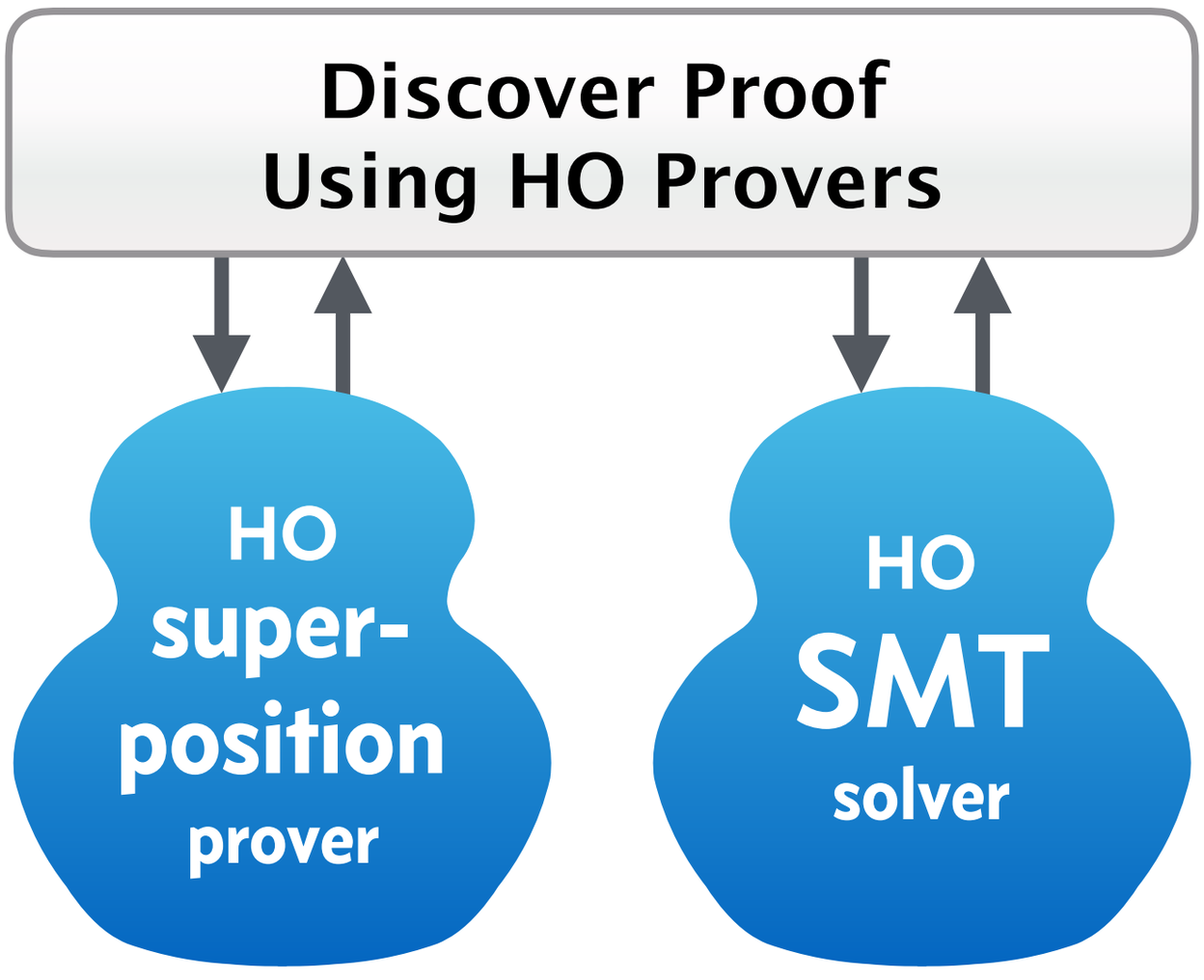
Push button automation for proof assistants (e.g. Coq)
based on **efficient higher-order (HO) provers**



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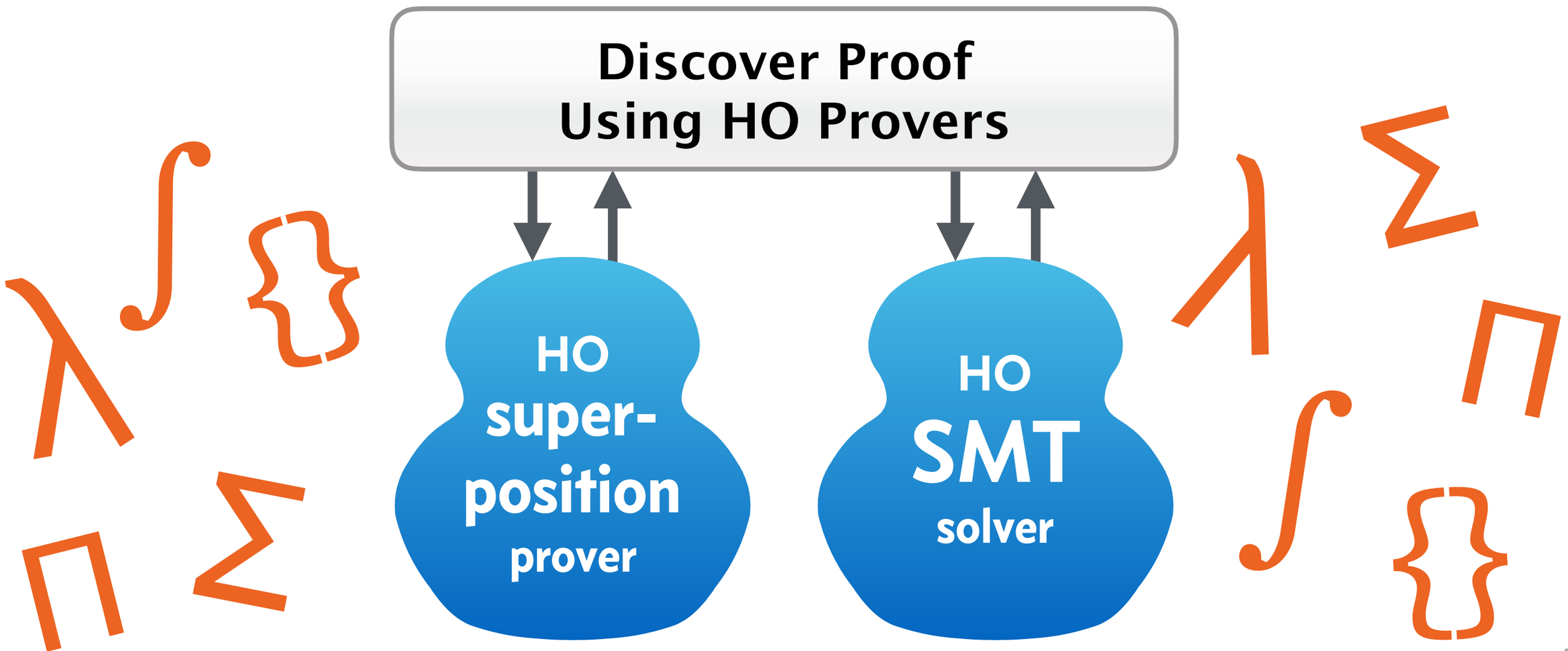

Application: A Verified "EasyChair"

- Induction Rule
- Simplifier
- Arithmetic Procedure
- General Reasoner
- First-Order Provers via Sledgehammer

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"PC members cannot review papers if they have a conflict of interest"

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fully automatic

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boilerplate

manual hints

fully automatic

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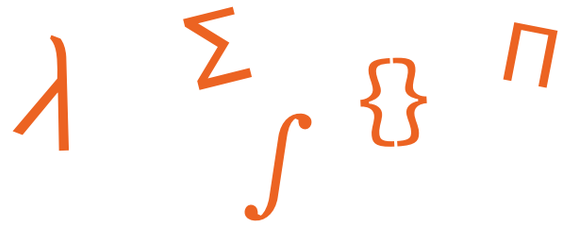
Discover Proof Using HO Provers



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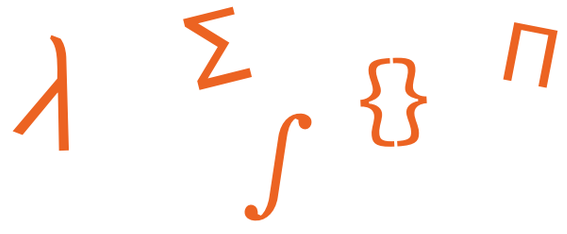

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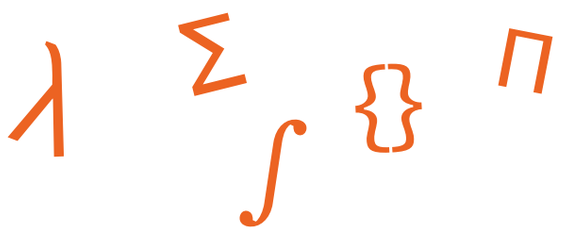

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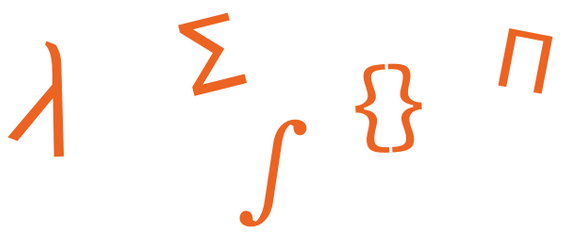

missing proof

fully automatic

My Grand Challenge

Create **efficient proof calculi** and **higher-order provers** targeting proof assistants and their applications to software and hardware development

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Scientific Objectives

SO1. **Extend superposition and SMT** to higher-order logic

SO2. Design practical **methods and heuristics** based on benchmarks

SO3. Conceive **stratified architectures** to build higher-order provers

SO4. **Integrate** our provers into proof assistants (Coq, Isabelle, TLA$^+$)

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SO1 — Higher-Order Superposition (λSUP)

First-order rule:

$$\frac{D' \vee t \approx t' \qquad C' \vee s[u] \not\approx s'}{(D' \vee C' \vee s[t'] \not\approx s')\sigma}\ \text{SUP-Left}$$

where $\sigma = \mathrm{mgu}(t, u)$ u is not a variable $t\sigma \not\trianglelefteq t'\sigma$ $s\sigma \not\trianglelefteq s'\sigma$
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- We need **sequences of unifiers**
- We need **higher-order term ordering**

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- We need **sequences of unifiers**
- We need **higher-order term ordering**
- We also want **proof-assistant-style HO rewriting**

SO1 — Higher-Order Term Orderings

	First-order	
	KBO	LPO
Well-foundedness	✔	✔
Transitivity	✔	✔
Stability under substitution	✔	✔
FO subterm property	✔	✔
Compat. with FO contexts	✔	✔
Totality for ground terms	✔	✔

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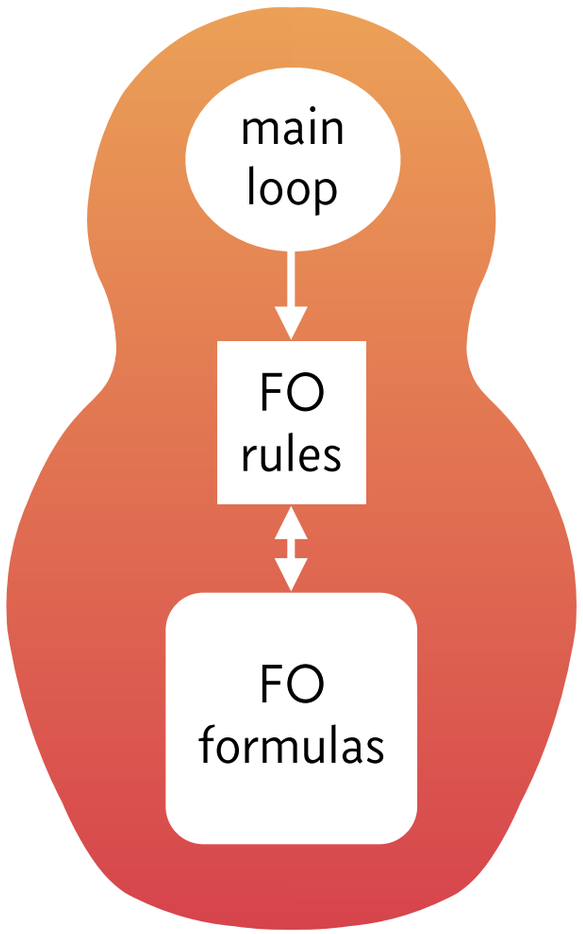
	First-order		Higher-order
	KBO	LPO	CPO/ HORPO
Well-foundedness	✔	✔	✔
Transitivity	✔	✔	✘
Stability under substitution	✔	✔	✘
FO subterm property	✔	✔	?
Compat. with FO contexts	✔	✔	✔
Totality for ground terms	✔	✔	✘
HO subterm property			?
Compat. with HO contexts			✔

SO1 — Higher-Order Term Orderings

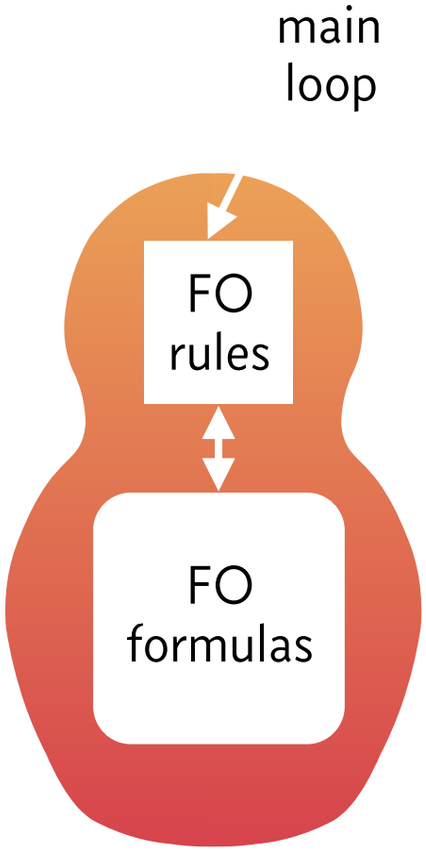
	First-order		Higher-order		
	KBO	LPO	CPO/ HORPO	λfKBO	λfLPO
Well-foundedness	✔	✔	✔	✔	✔
Transitivity	✔	✔	✘	✔	✔
Stability under substitution	✔	✔	✘	✔	✔
FO subterm property	✔	✔	?	✔	✔
Compat. with FO contexts	✔	✔	✔	✔	✔
Totality for ground terms	✔	✔	✘	✔	✔
HO subterm property			?	✔	✔
Compat. with HO contexts			✔	✔	(✔)

SO3—Stratified Architecture



First-Order Prover (e.g. veriT)

SO3 — Stratified Architecture



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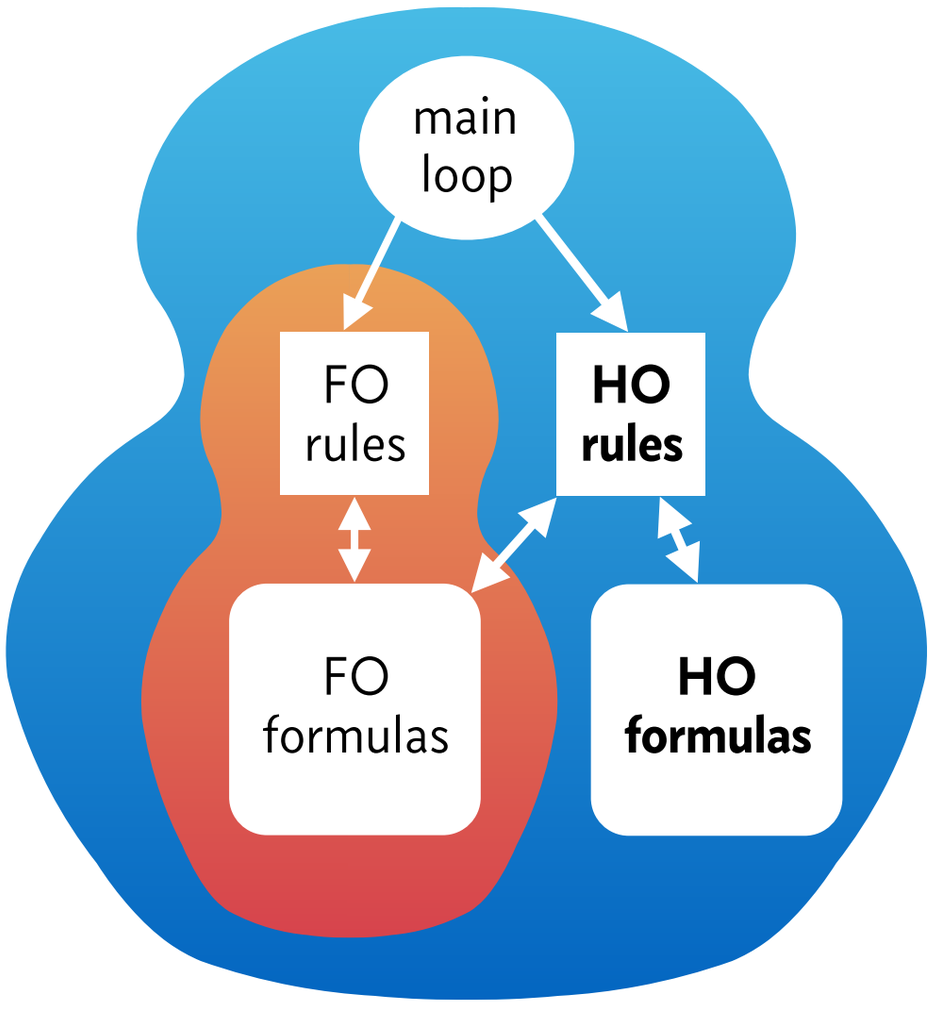

First-Order Prover (e.g. veriT)
Matryoshka Prover (e.g. veriHOT)

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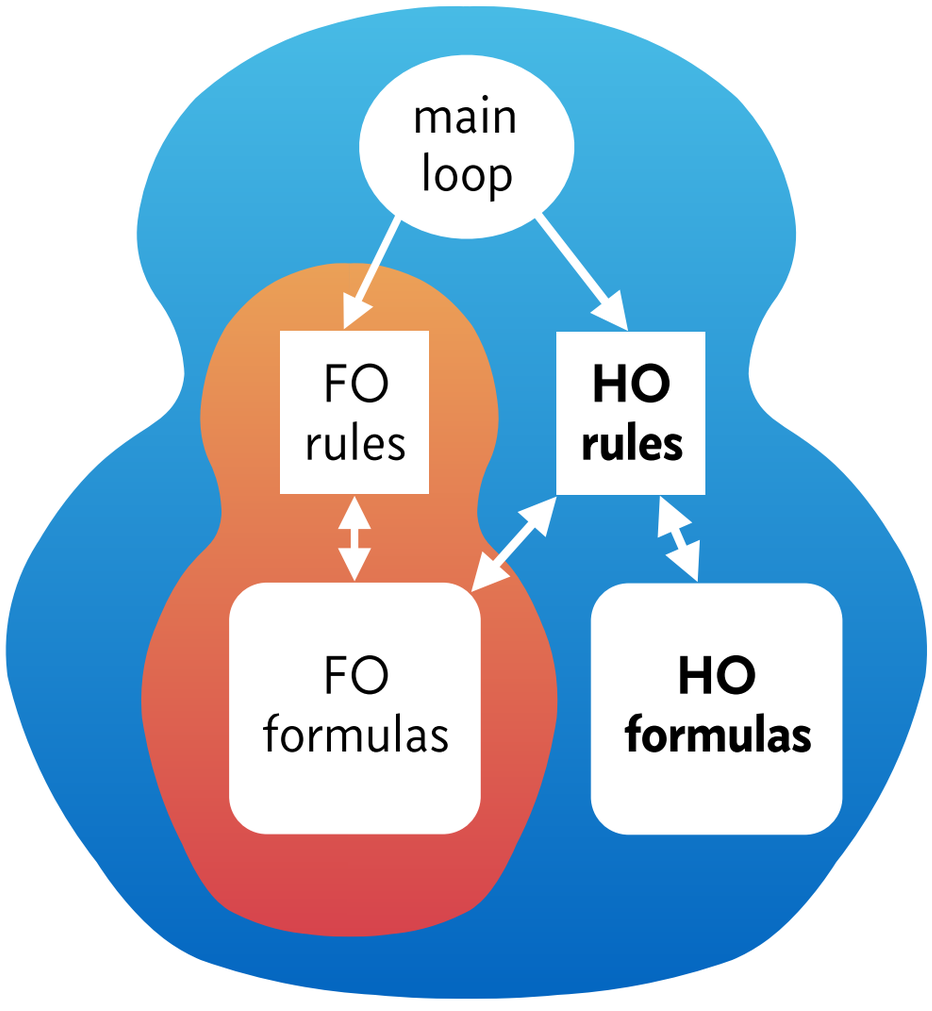

Inspired by Nelson–Oppen (SMT)

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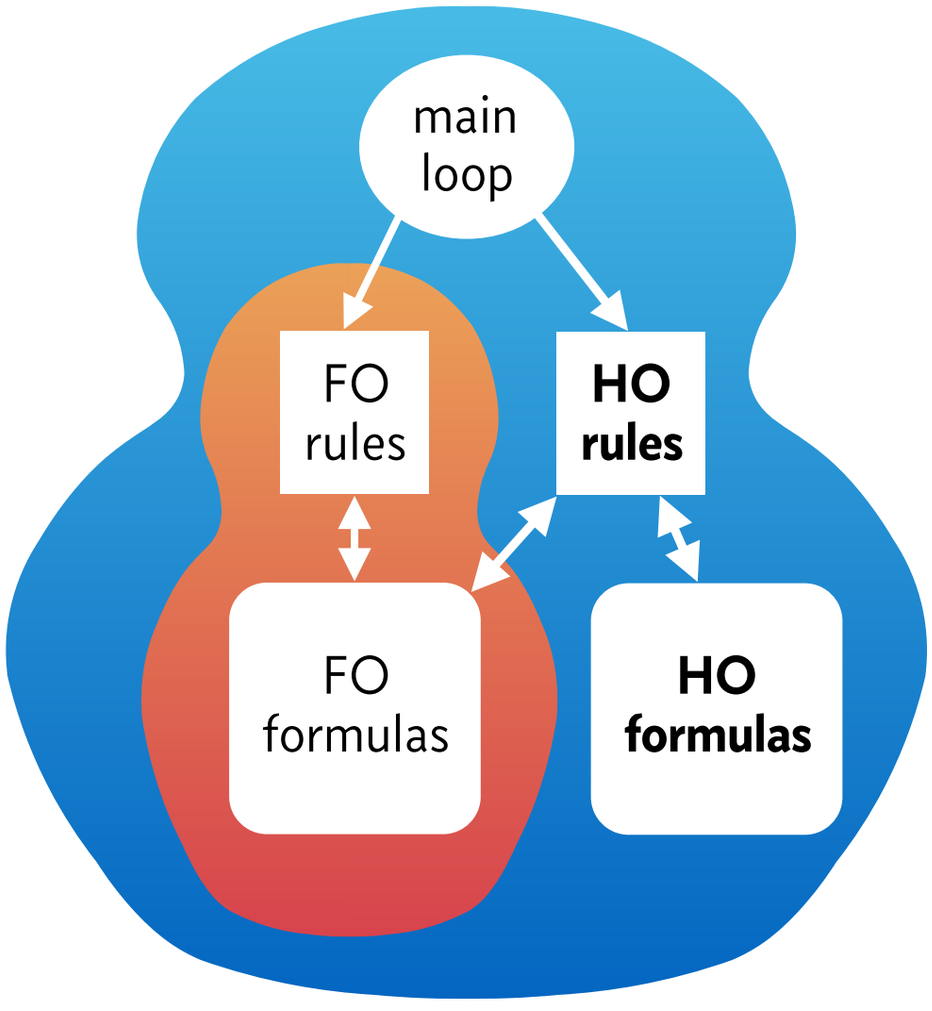

Inspired by Nelson–Oppen (SMT)

Base FO provers: SPASS & veriT

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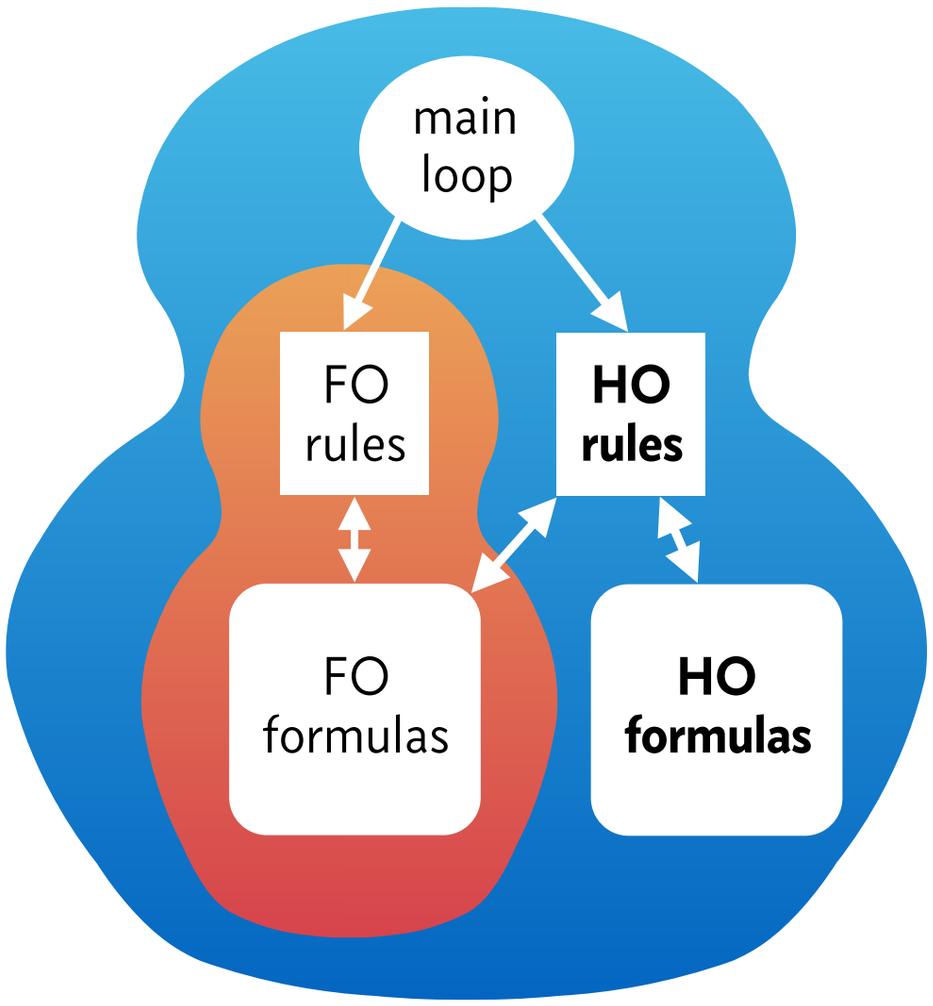


Inspired by Nelson–Oppen (SMT)

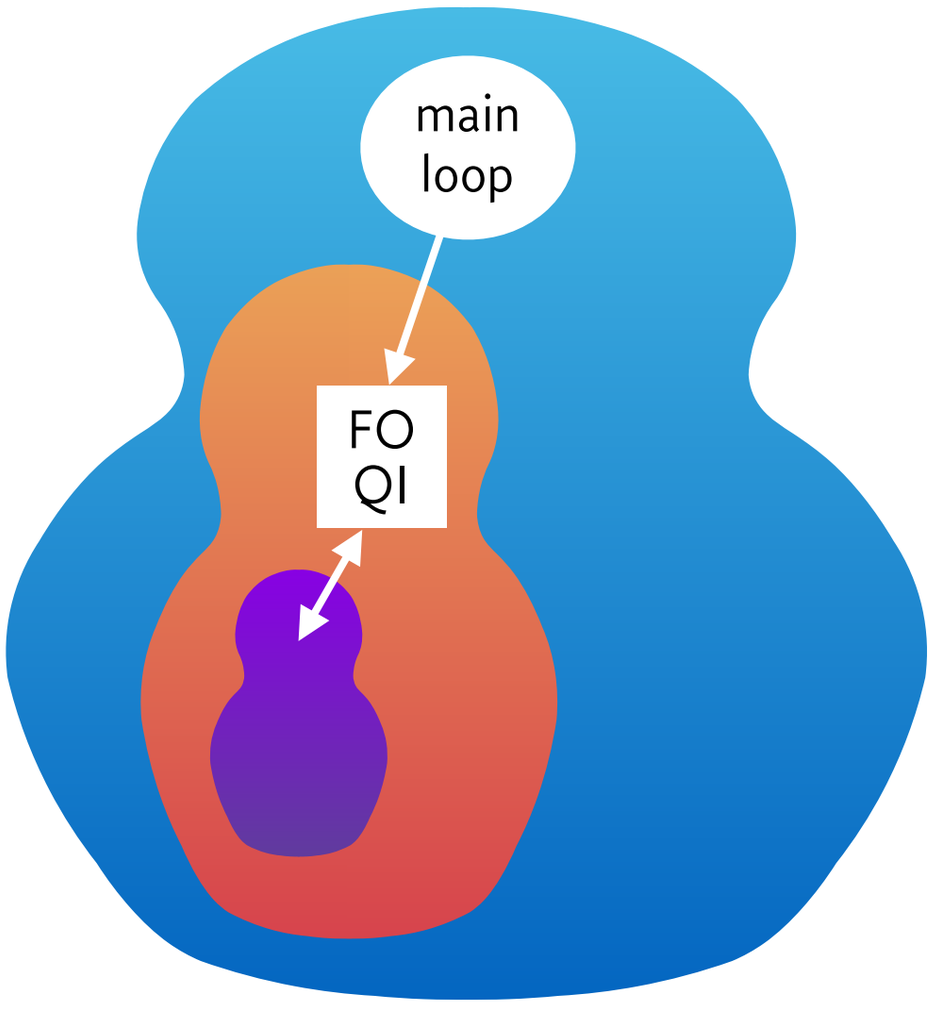
Base FO provers: SPASS & veriT

Some scientific challenges:

- How to exploit derived FO formulas and/or candidate models to guide HO quantifier instantiation?
- How to generate certificates for reconstruction in proof assistants?

First-Order Prover (e.g. veriT)
Matryoshka Prover (e.g. veriHOT)

SO3 — Higher-Order SMT



First-order quantifier instantiation (QI):

- E-matching (triggers)
- Model-based QI
- Conflict-guided instantiation
- Congruence closure with free variables

SAT Solver (e.g. MiniSat)
First-Order Prover (e.g. veriT)
Matryoshka Prover (e.g. veriHOT)

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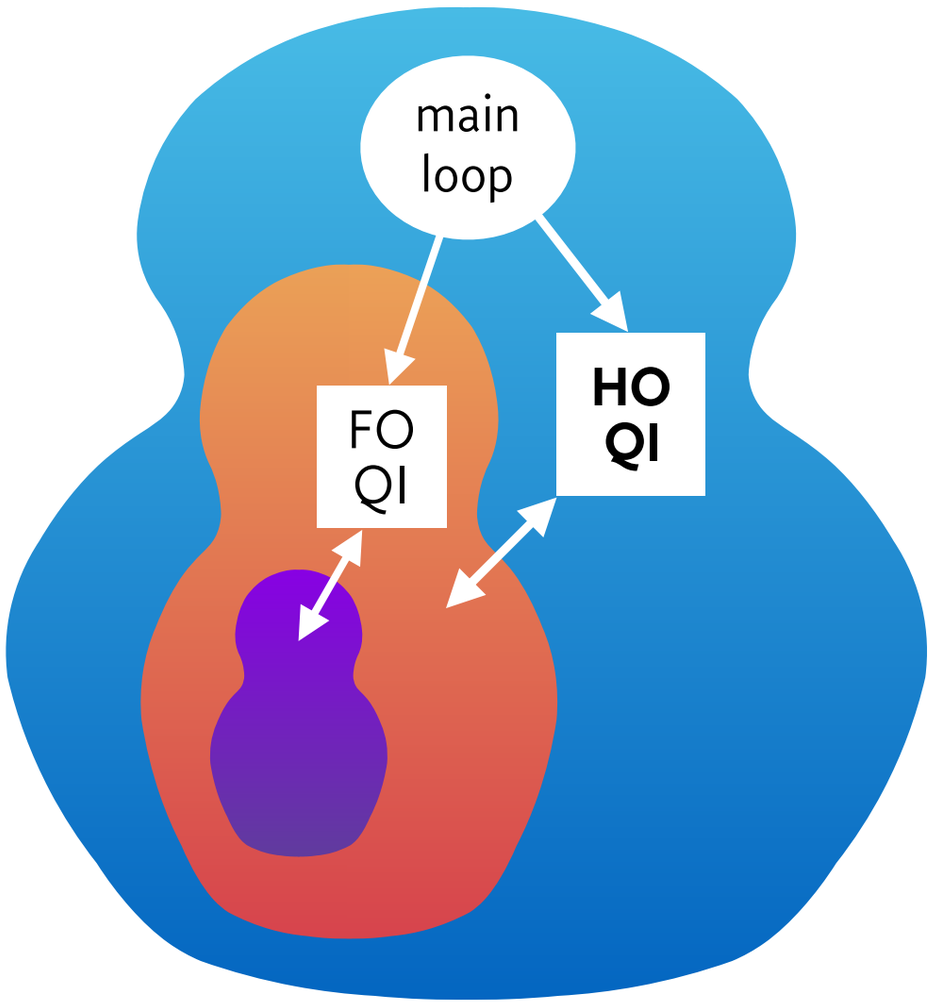

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- E-matching (triggers)
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- We need to **extend these strategies to higher-order logic**

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First-Order Prover (e.g. veriT)
Matryoshka Prover (e.g. veriHOT)

SO4—Connection with Proof Assistants

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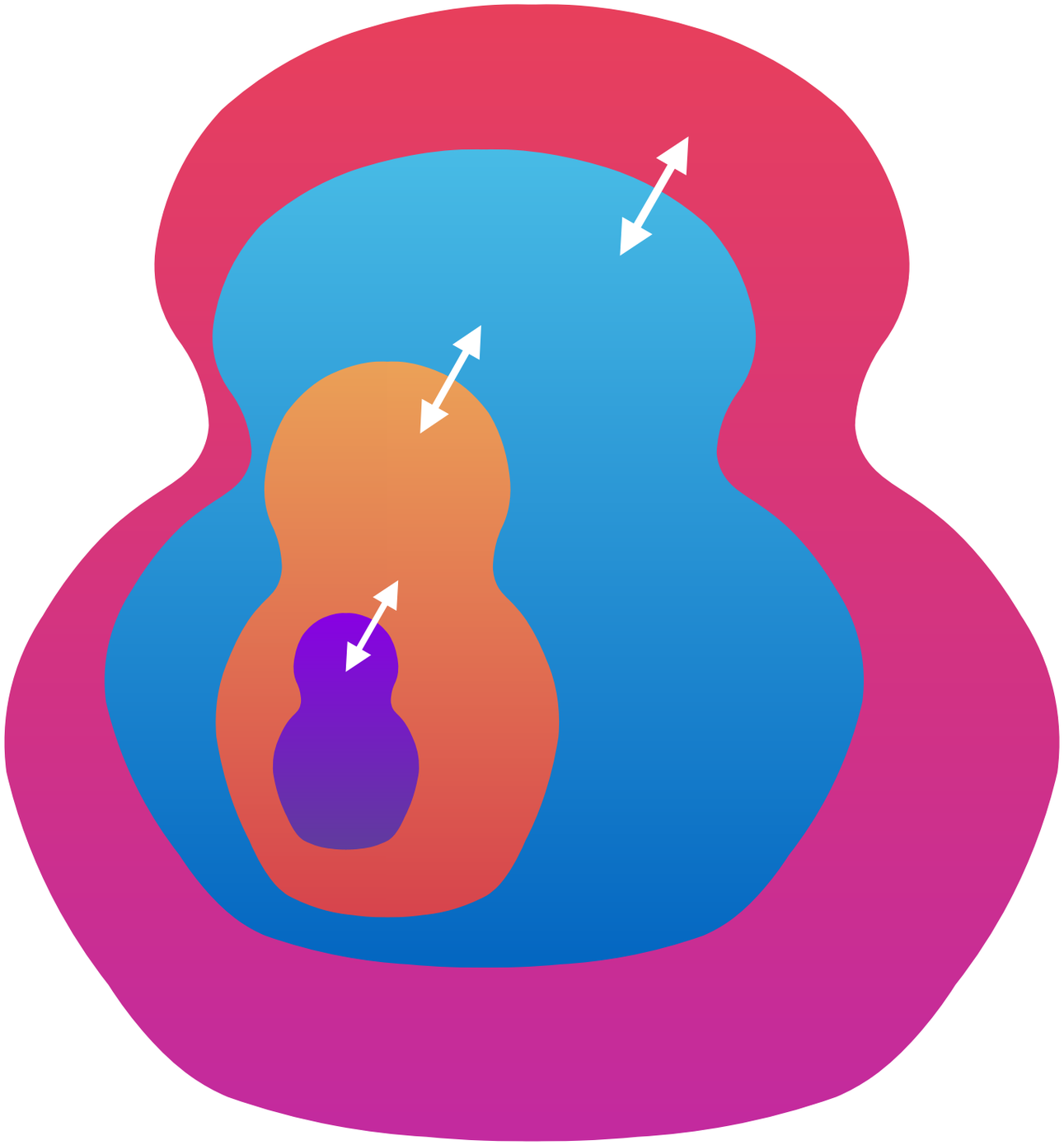
First-Order Prover (e.g. veriT)
Matryoshka Prover (e.g. veriHOT)
Proof Assistant (e.g. Coq)

SO4—Connection with Proof Assistants



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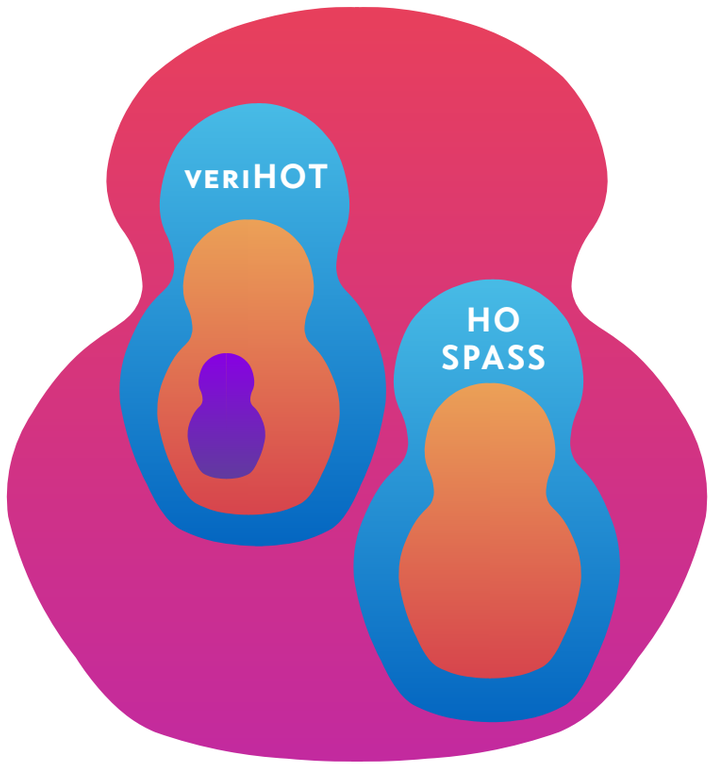
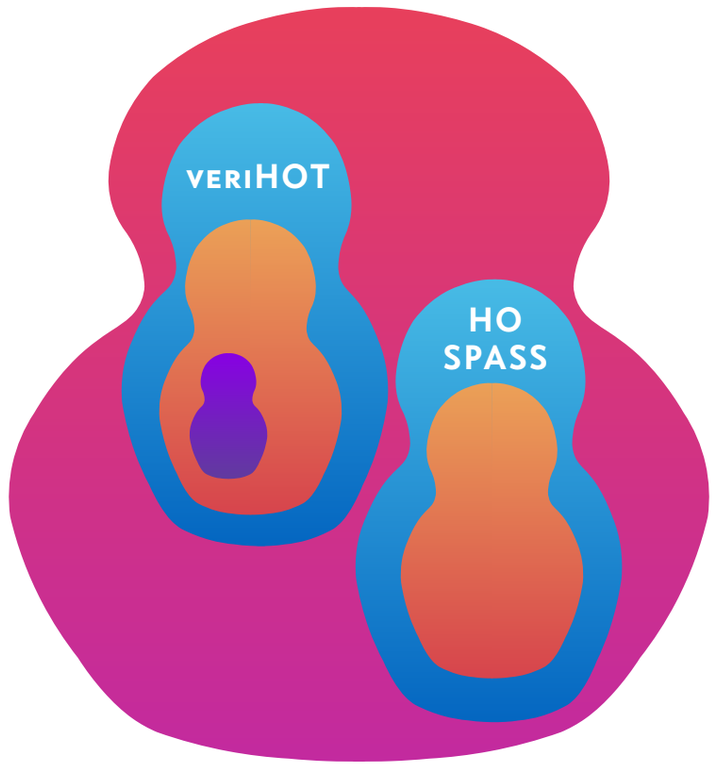
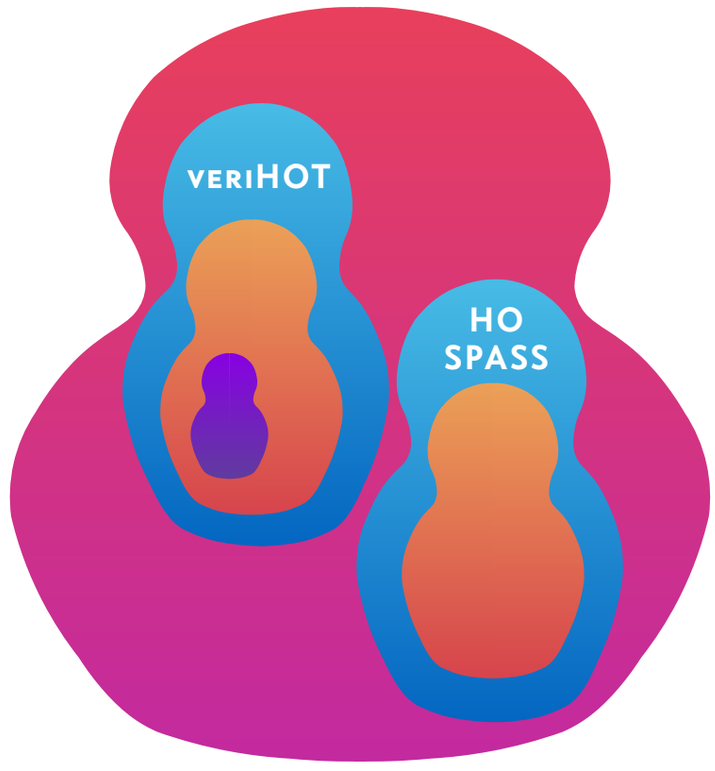
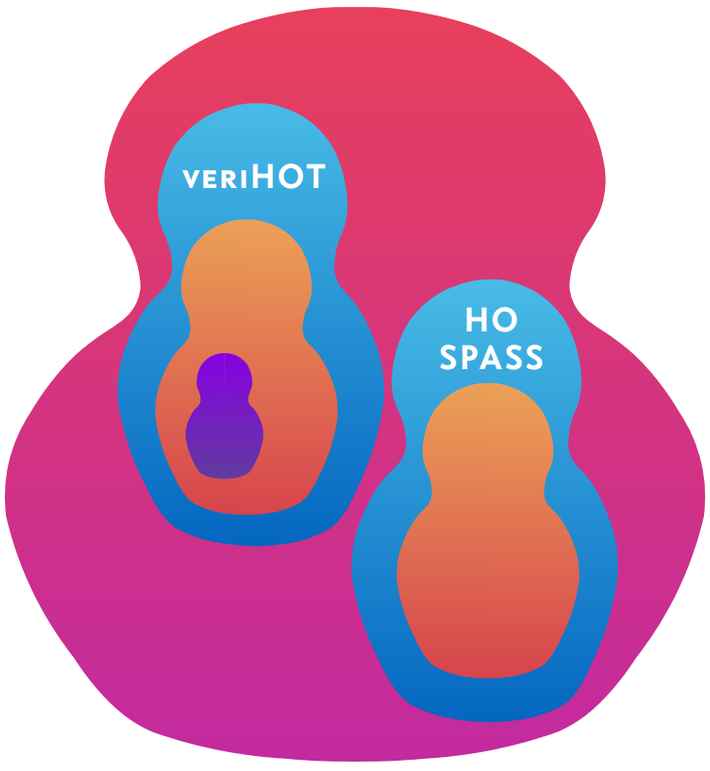






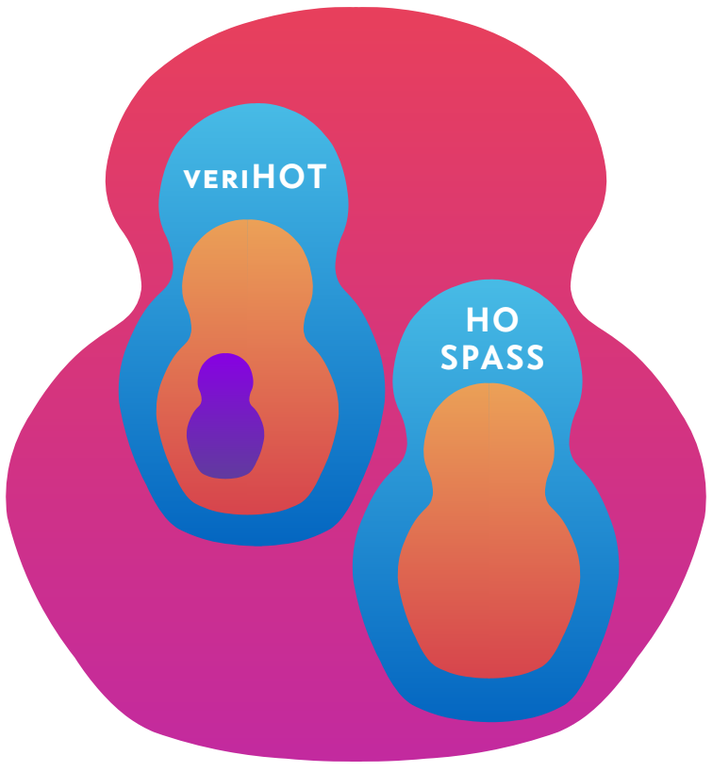
SO4—Connection with Proof Assistants

Dependent
Type Theory



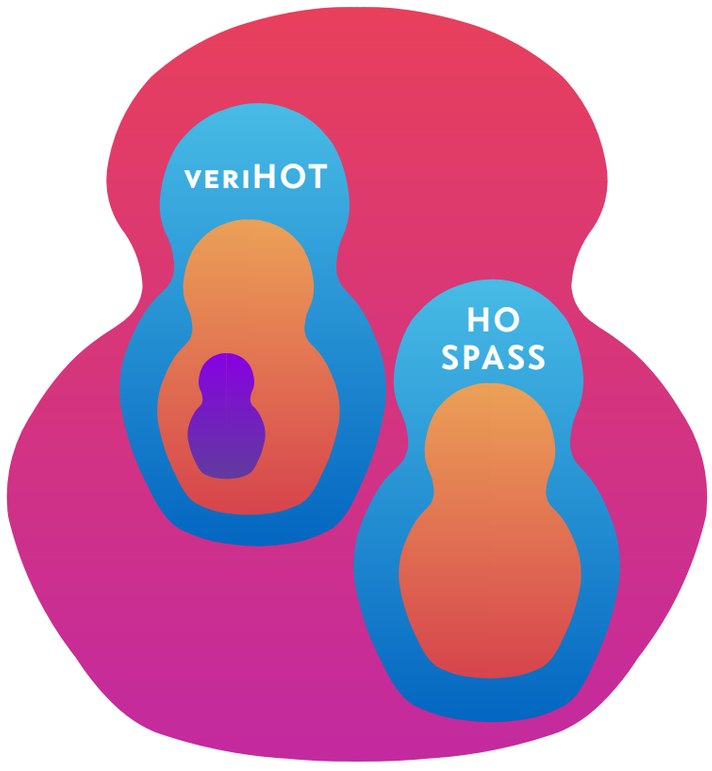
Coq

Classical
Higher-Order Logic



Isabelle/HOL

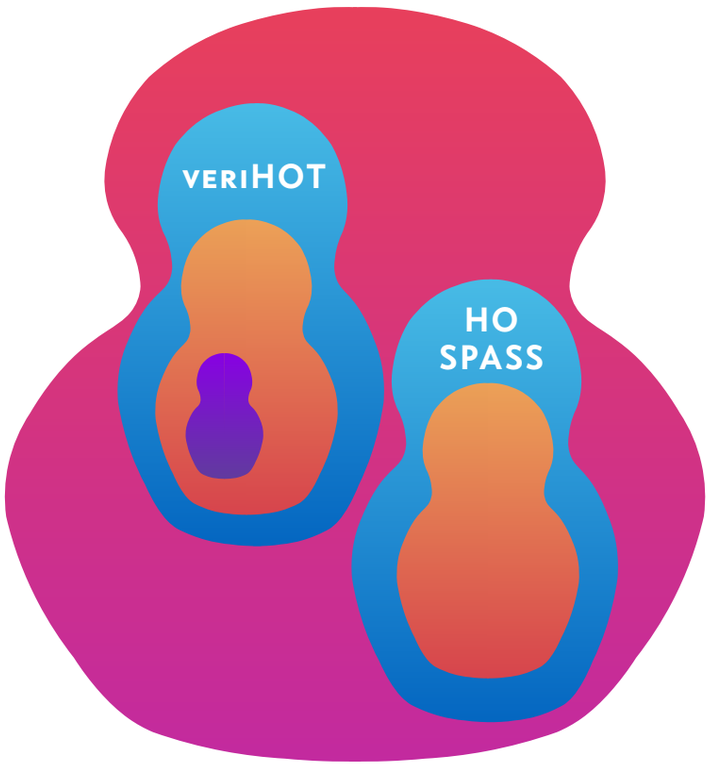
Set Theory



TLA+

SO4—Connection with Proof Assistants

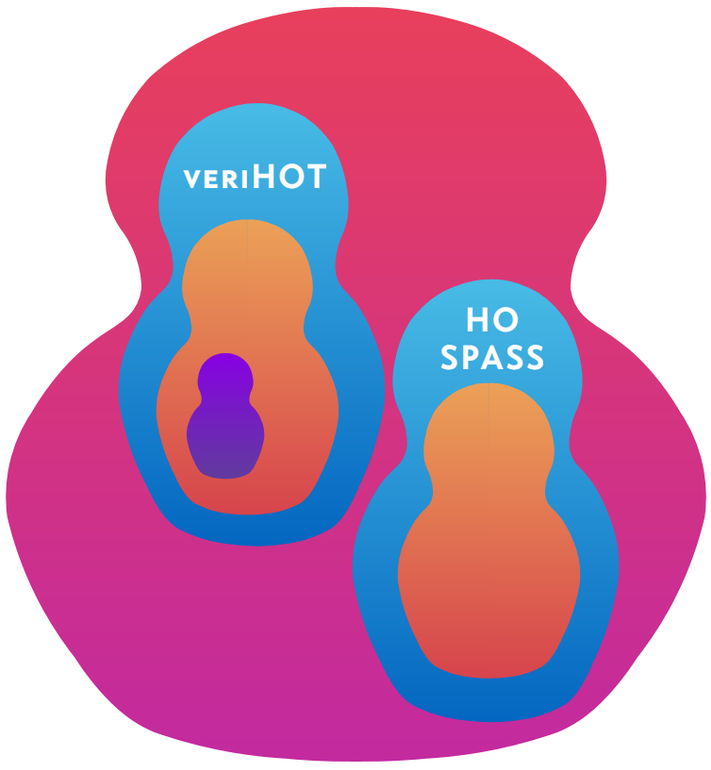
Dependent
Type Theory



Coq

Agda
Lean
Matita
⋮

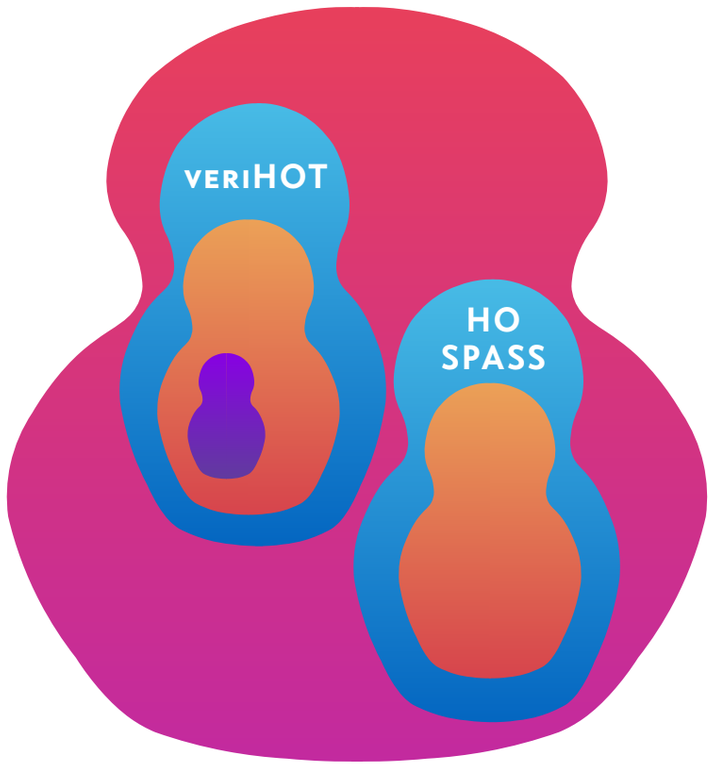
Classical
Higher-Order Logic



Isabelle/HOL

HOL4
HOL Light
PVS
⋮

Set Theory



TLA+

Isabelle/ZF
Mizar
Rodin (Event-B)
⋮

The Matryoshka Research Group

Scientific Leader: J. C. Blanchette (75%)

SMT & veriT — *Permanent Member:* Pascal Fontaine (20%)

Higher-order reasoning — Junior Researcher

Superposition — Ph.D. Student 1

SMT — Ph.D. Student 2

Proof assistants — Ph.D. Student 3

Implementation — Software Engineer

Superposition & SPASS — *External Collaborator:* Uwe Waldmann (MPII)

Visiting Researchers:

- developers of automatic provers
- developers and users of proof assistants

Matryoshka in one Slide

Grand Challenge & Outcome

Create efficient **proof calculi** (λSUP & λSMT) and stratified **higher-order provers** (HO SPASS & veriHOT) to dramatically **improve automation** in proof assistants

Risks

- Efficient HO automation is a long-standing open problem
- Proposed stratified architecture has never been tried

Impact

- The project will recast the **methods of automatic proving** to reach the **goals of interactive proving**
- Interactive verification will become a **cost-effective** option for building **software & hardware** of the **highest quality**