Hints for AVATAR (and some more)

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“Interactive Theorem Proving” with ATPs

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- targeting open problems (e.g. the AIM conjecture)

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- a single proof attempt (ATP call) usually does not solve it
- trying different formulations / axiomatizations
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➥ By the way, these attempts may run for weeks!

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What is a hint?

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How to come up with hints automatically?

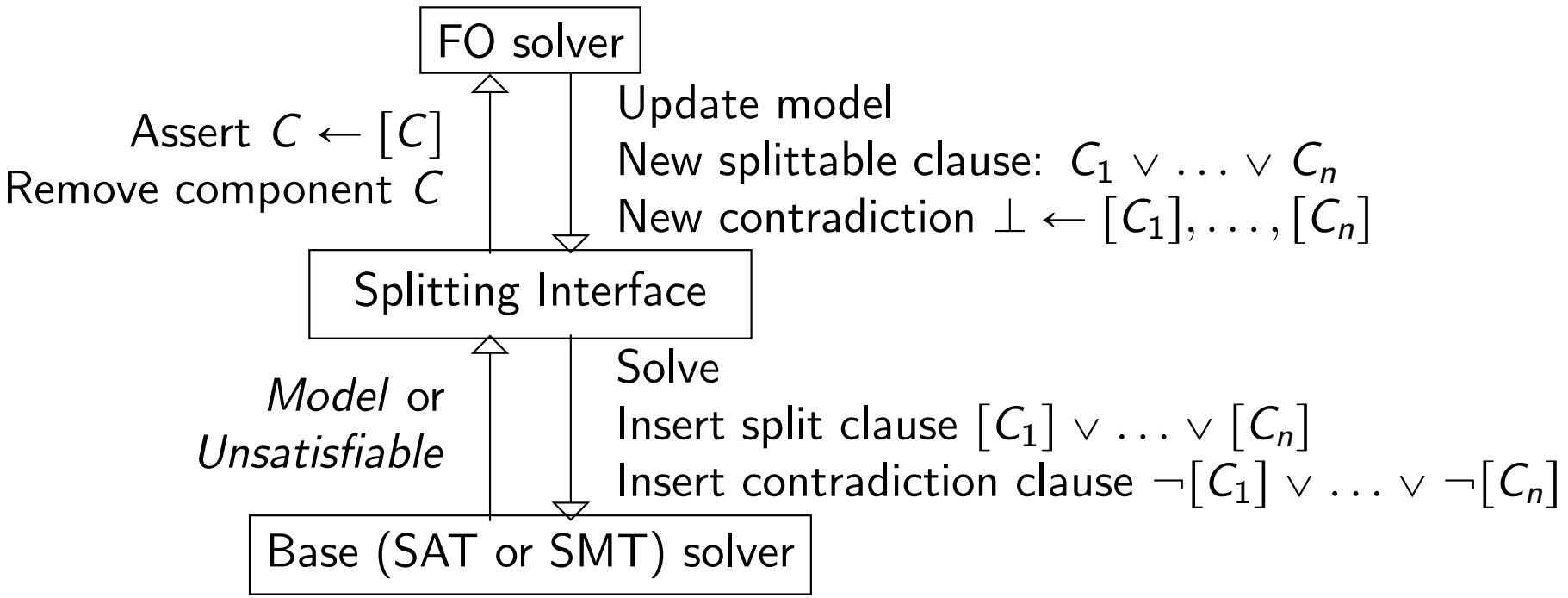
AVATAR: a reminder

AVATAR [Voronkov'14]

- modern architecture of first order theorem provers
- integrates saturation with a SAT solver (or an SMT solver)
- efficient realization of the *clause splitting rule*
- instead of one monolithic proof search
  a sequence of proof searches on (much) smaller sub-problems

- implemented in theorem prover Vampire
- shown highly successful in practice

AVATAR architecture overview



Boosting AVATAR with hints

Instead of waiting for the user to supply hints for problem P . . .

. . . attempt P using AVATAR and collect as hints the first-order parts of the clauses appearing in the sub-proofs of the so far derived contradiction clauses

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DEMO!

Outline

1. Hints for AVATAR
2. An Experiment
3. What is a Significant Improvement?

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1 Hints for AVATAR

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3 What is a Significant Improvement?

Experimental setup

Vampire setup:

- `--saturation_algorithm discount` (for stability)
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- `--time_limit 10` (reasonable time to finish)

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The benchmark:

- TPTP v 7.2.0
- 17573 eligible first-order problems

Results

(on Starexec)

configuration	solved	uniques	additional
base	7914	0	7914
base+hints	7882	2	62
sac	8100	13	299
sac+hints	8106	13	23

base = `-sa discount -awr 10 -t 10`

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Experimented with AVATAR flushing; also not very interesting

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(30 problems is approx. 0.5‰ of the benchmark size)

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➥ Ongoing and future work!

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   ➥ To have a chance to improve strategy schedule . . .

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Let's have some random fun!

Randomly permuting the input problem

Use `tptp4X -trandomize` from the TPTP toolset to:

- randomize the order of commutative logical operations
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➥ recalling **randoCoP (Raths, Otten; 2008)**

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Clause Selection and Age-weight Ratio

Vampire alternates between selecting the next given clause by age (old first) and by weight (light first) under a given ratio.

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base	8725	12	8725
rnd1	8747	8	91
rnd2	8744	16	37
rnd3	8768	23	37
rnd4	8735	14	21
rnd5	8741	16	16

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Why don't we use (carefully seeded) randomness to prove more theorems (without much actual extra thinking)?