

# Symbolic Regression Using Prior Knowledge

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# Symbolic Regression Using Prior Knowledge

## Insufficient training data

- sparse and noisy,
- unevenly sample the input space,
- may completely omit some parts of the input space.

## Models trained using only such training data tend to be

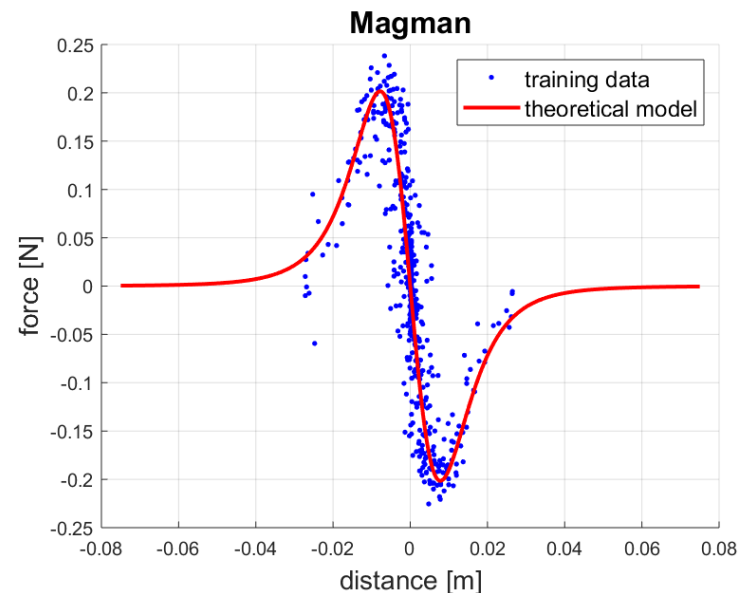
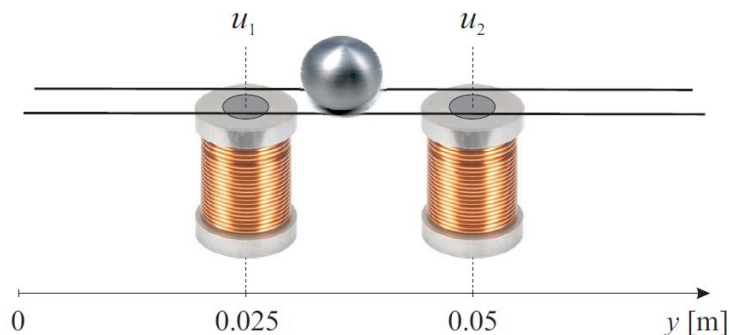
- overfitted,
- partially incorrect in terms of their steady-state characteristics or local behavior.

# Magnetic manipulation

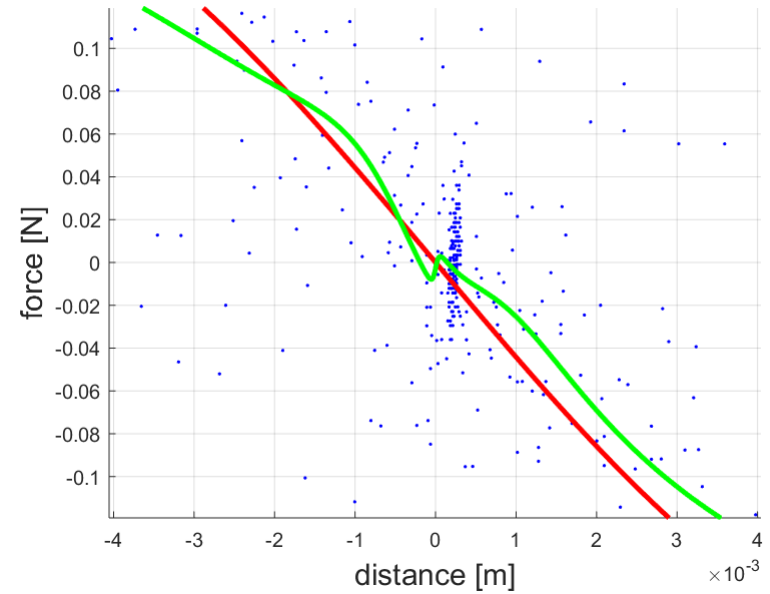
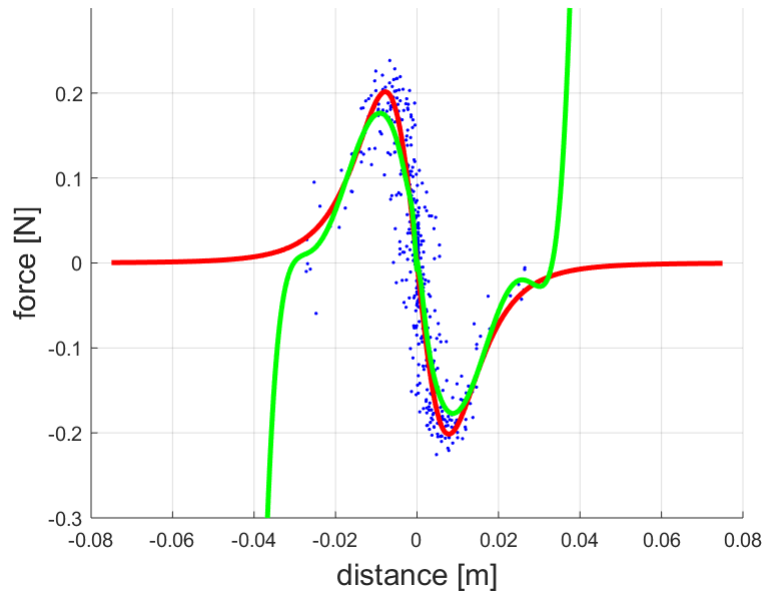
**Magnetic manipulation** – an iron ball moving along a rail and an electromagnet at a static position under the rail.

**Data** – noisy; only a part of the input space is covered.

**Goal** is to find a model of the nonlinear magnetic force affecting the ball as a function of the distance between the ball and the activated coil.



# Magman: SR driven by training data only



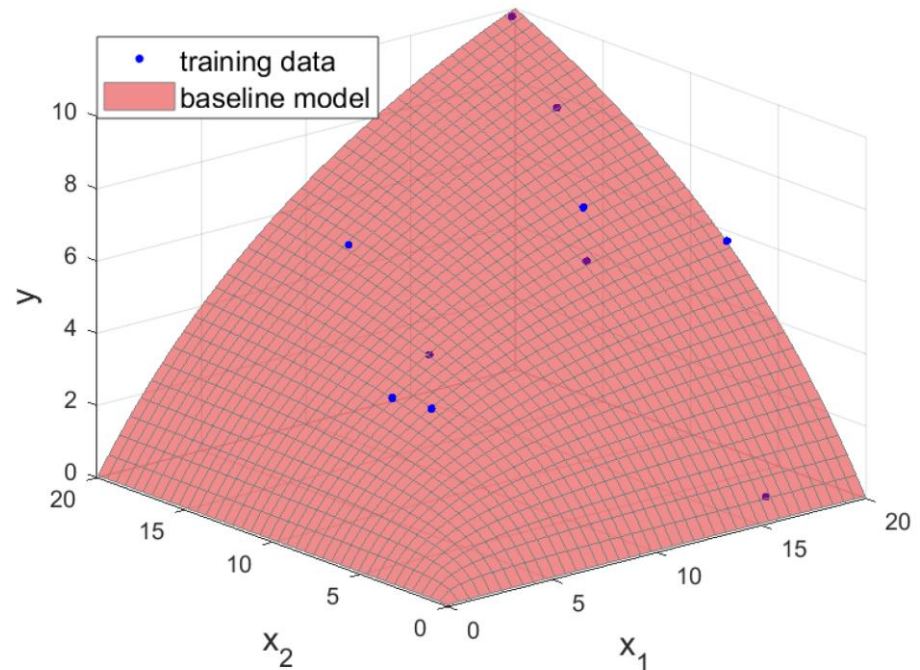
# Two resistors in parallel

**Resistance** – equivalent resistance of two resistors in parallel.

**Data** – very sparse and noisy.

**Goal** is to find a model that fits the data and obeys the physical law.

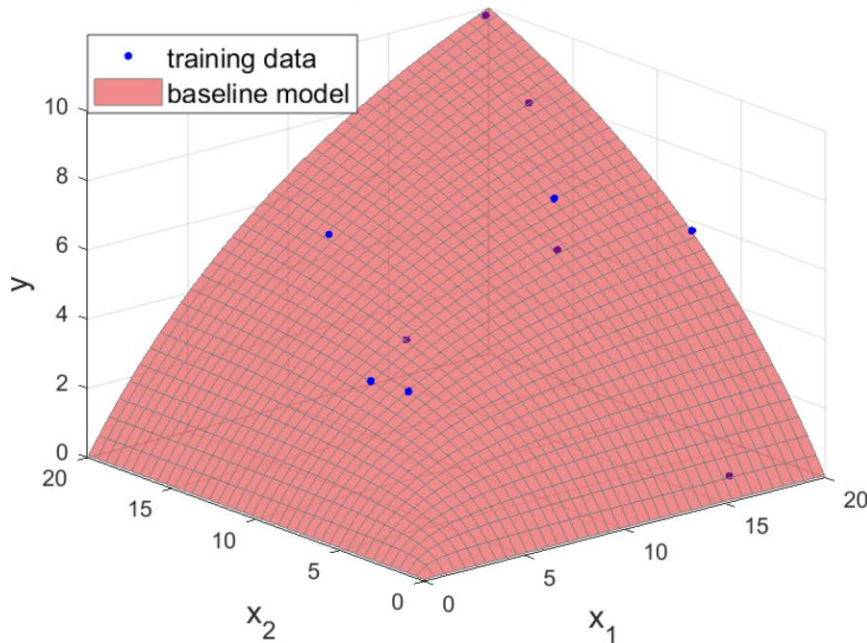
Baseline model:  $R = \frac{R_1 R_2}{R_1 + R_2}$



# Resistance: SR driven by training data only

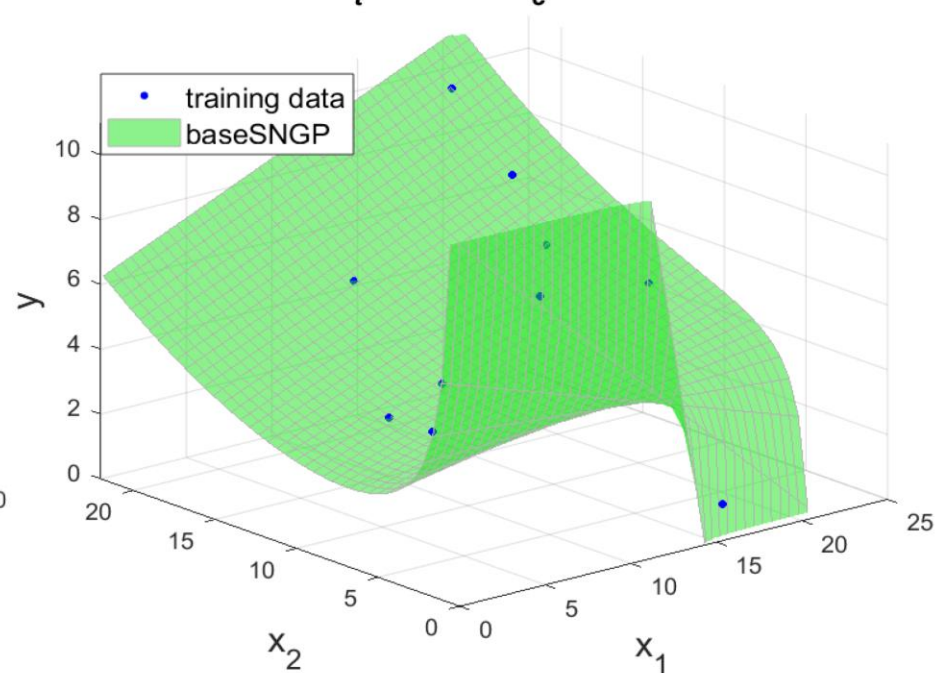
## Baseline model

$$C_t = 0.087, C_c = 0$$



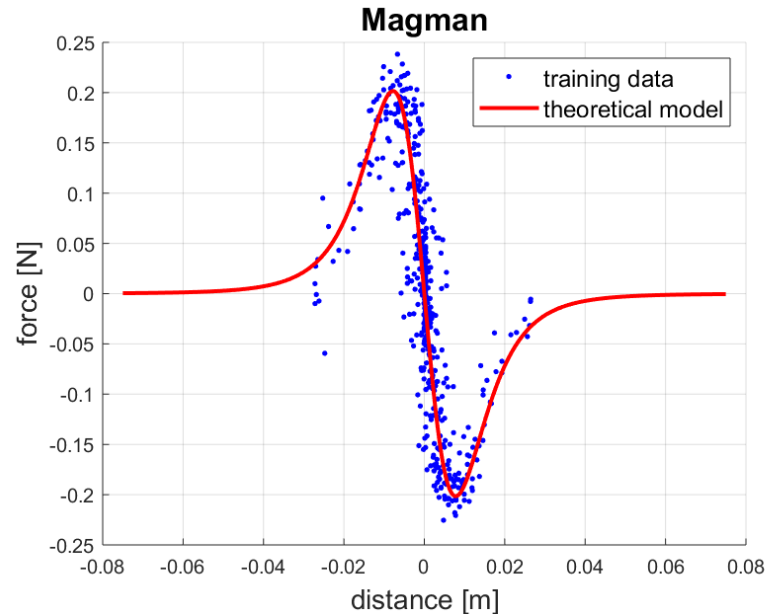
## SR model

$$C_t = 0.013, C_c = 9200$$



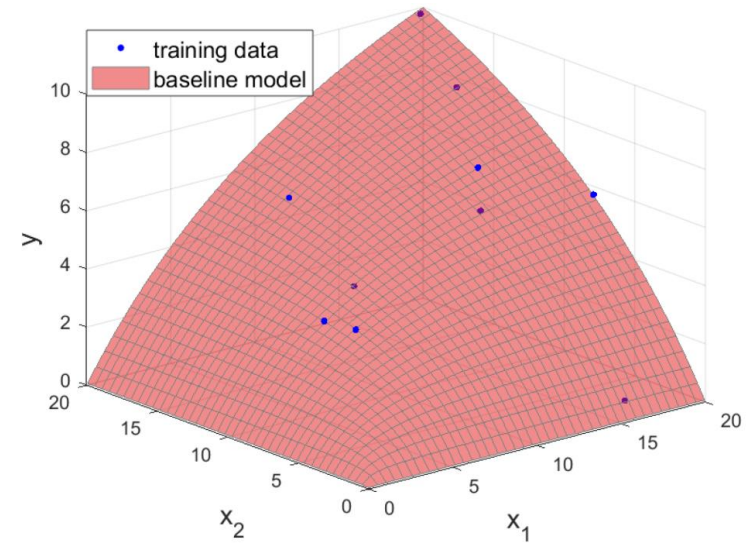
# Magman: Desired model's properties

- Increasing monotonicity  
 $x \in (-0.075, -0.01)$   
or  
 $x \in (0.01, 0.075)$
- Decreasing monotonicity  
 $x \in (-0.01, 0.01)$
- Odd symmetry
- Exact output values  
 $f(-0.075) = 0.001$   
 $f(0.075) = -0.001,$   
 $f(0) = 0.0$



# Resistance: Desired model's properties

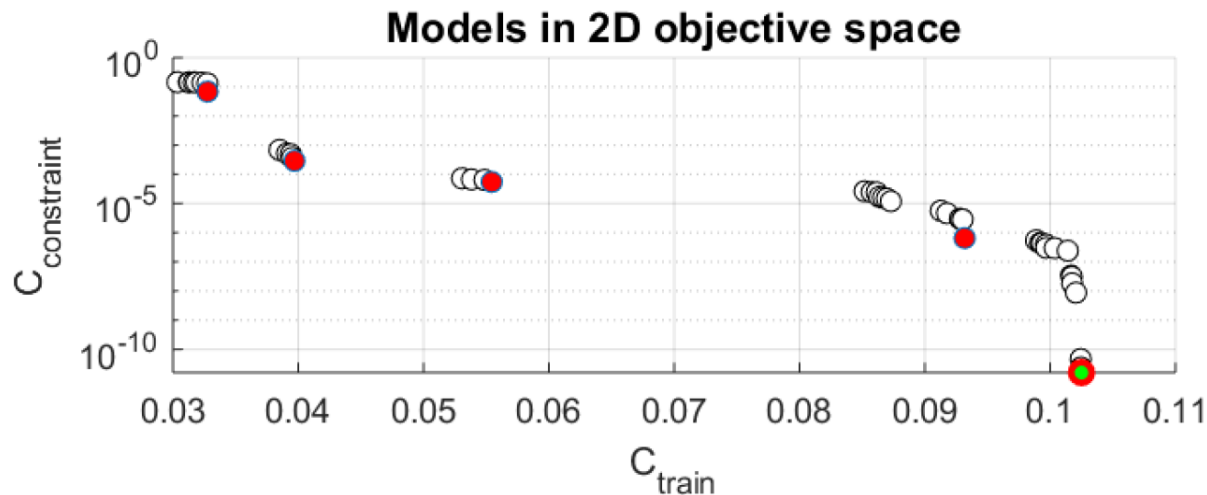
- symmetry with respect to arguments  
 $R(R_1, R_2) = R(R_2, R_1)$
- domain-specific constraint  
 $R_1 = R_2 \Rightarrow R(R_1, R_2) = R_1/2$
- domain-specific constraint  
 $R(R_1, R_2) \leq R_1, R(R_1, R_2) \leq R_2$





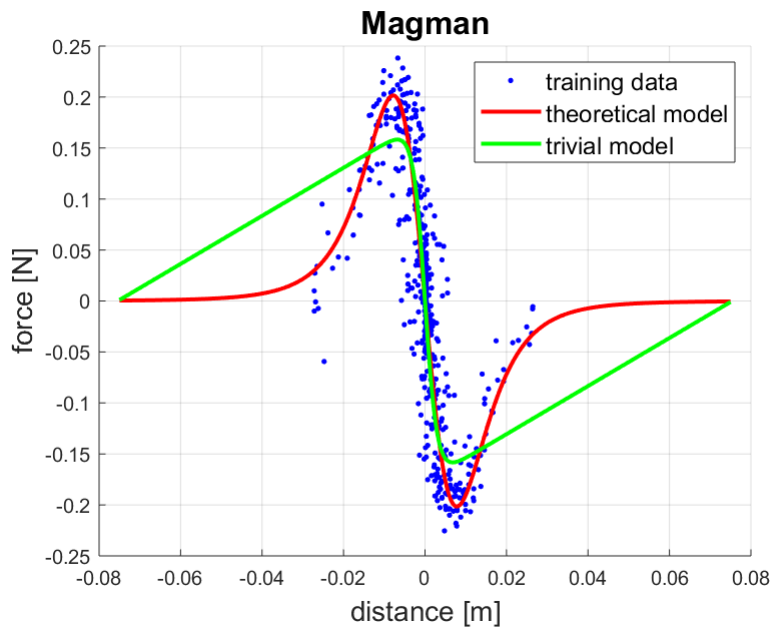
# Bi-objective Symbolic Regression

- Optimisation criteria
  - minimise prediction error on training data samples
  - minimise violation of the desired model's properties
- Constraint samples set – properties are internally represented by a set of discrete data samples on which candidate models are exactly checked.
- NSGA-II – based on the concept of dominance
  - generates a set of non-dominated solutions

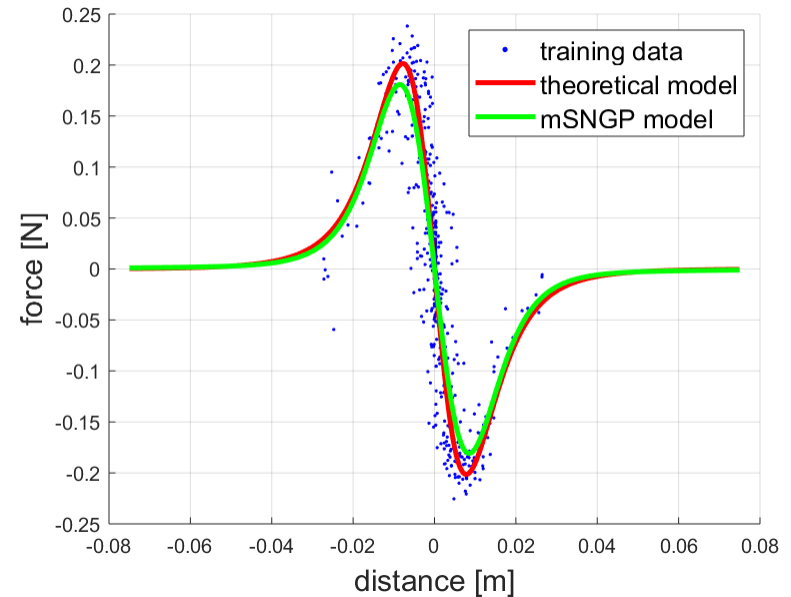


# Bi-objective SR: Magman

Inaccurate, but perfectly valid

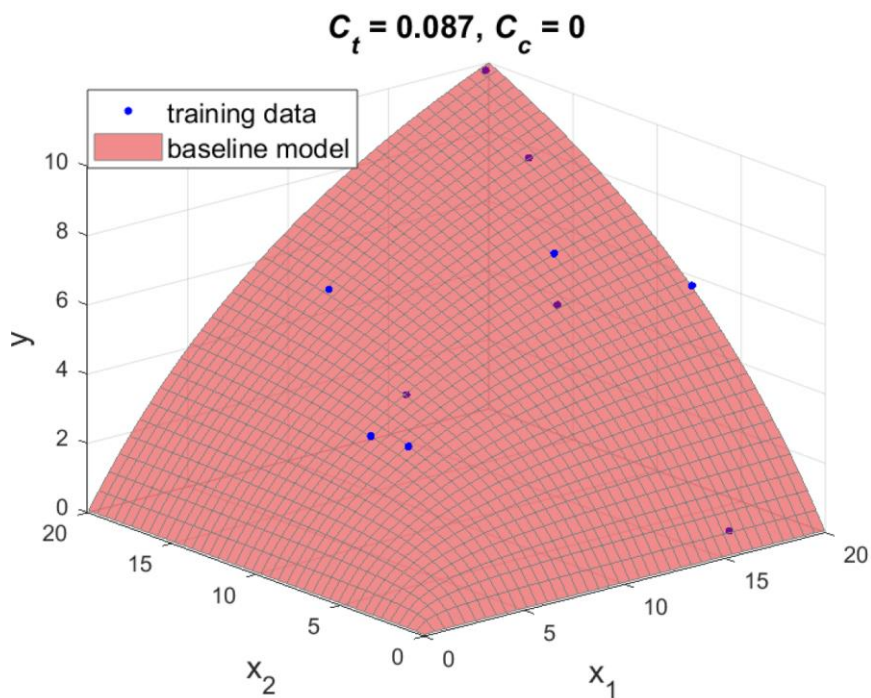


Accurate and valid

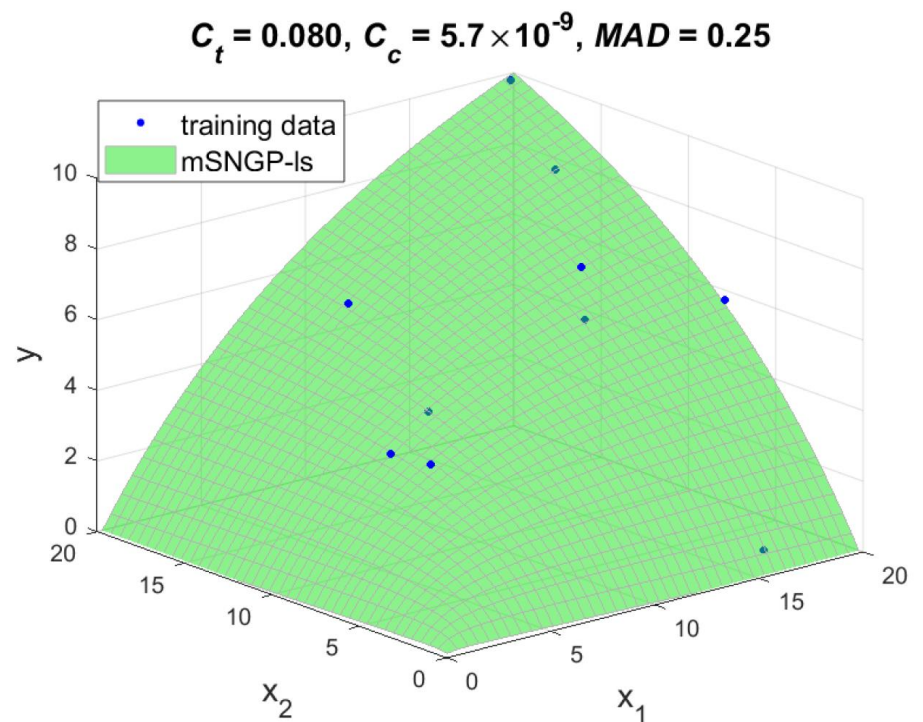


# Bi-objective SR: Resistors

## Baseline model



## SR model



# Summary

- Multi-objective SR method that produces realistic models that fit well the training data while complying with the prior knowledge of the desired model characteristics at the same time.
- Future work
  - Investigate various strategies to maintain the most relevant constraint samples during the whole run.
  - Different constraints can generate violations of a very different scale – need for some normalization.